# Longitudinal Autopilot Design

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Longitudinal autopilot controllers were designed for a vintage propeller transport aircraft as well as a jet transport aircraft. The controls were then tested against different input disturbances with the use of Simulink.

#### Nomenclature

- k = gain of amplifier
- s = transfer function's imaginary frequency
- $\Theta$  = Laplace of pitch angle
- $\Delta$  = Laplace of elevator deflection

## I. Introduction

IN this System Dynamics and Controls term project, the individual students were to design longitudinal autopilot controllers for both propeller and jet aircraft given the respective linearized aircraft and servo dynamics.

The piston aircraft was a vintage propeller transport aircraft that was frequently ferried to air shows. The pilots wanted the opportunity to serve themselves food and non-alcoholic beverages on these flights, and so requested an autopilot be installed to operate at cruise conditions (20,000 ft and 210 knots). Due to the pilots' budget constraints, only a vertical gyro and an adjustable amplifier could be added to the plane.

The jet aircraft's longitudinal controller was to be designed for operation at 40,000 ft and 470 knots. Due to the additional complexity associated with a jet aircraft, a rate gyro could be used along with a vertical gyro and amplifier.

#### **II.** Piston Aircraft

The linearized flight dynamics at the given ferrying conditions is:

$$\frac{\Theta(s)}{\Delta_F(s)} = \frac{-(s+3.1)}{s(s^2+2.8s+3.24)} \qquad \text{Eq. (1)}$$

Also, the servo can be modeled by the transfer function:

$$\frac{-1}{s+125}$$
 Eq. (2)

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Figure 1. Shown above is the block diagram for the completed piston aircraft autopilot.

Solving this block diagram for the yields a system transfer function of:

$$\frac{k(s+3.1)(s+12.5)(s)(s^2+2.8s+3.24)}{k(s+3.1)+(s+12.5)(s)(s^2+2.8s+3.24)}$$
Eq. (3)

Next, applying Routh-Hurwitz stability criterion to the system gives an adjustable amplifier or gain of -296.1 < k < 74.487. A gain of 9 was picked for its overall characteristics.



Figure 2. Autopilot with no disturbances. Reference is 10°.



Figure 3. Autopilot with a step disturbance. Reference is 10°.



Figure 4. Autopilot with ramp disturbance. Reference is 10°.

## **III.** Jet Aircraft

For the jet aircraft operating at 470 knots at 40,000 ft, the aircraft short-term dynamics from elevator deflection can be modeled by the following transfer function:

$$\frac{\Theta(s)}{\Delta_E(s)} = \frac{-1.39(s+0.306)}{s(s^2+0.805s+1.325)}$$
 Eq. (4)

The servo can be modeled with the transfer function given below.

$$\frac{-10}{s+10}$$
 Eq. (5)

These functions form a large part of the autopilot system which manipulates them in order to achieve level flight.



Figure 5. Block diagram for the jet longitudinal autopilot controller.

The transfer function for the system is a bit complicated, though easily found algebraically through Mason's rule. For simplicity, only the denominator, or the characteristic equation, is listed below.

$$\frac{(k_1k_2+k_2)(13.9s+4.2534)}{s^3+10.805s^2+9.375s+13.25}+1$$
 Eq. (6)

Applying the Routh-Hurwitz stability criterion to the characteristic eq. (6), yields eq. (7), shown below.

 $k_2(k_1+1) > 3.115$  Eq. (7)

Arbitrarily choosing a second gain of 2 means that the first gain needs to be at least greater than approximately 0.56. A k2 of 7 was ultimately chosen since it seemed to react the best to disturbances.



Figure 6. The longitudinal jet autopilot shown without disturbance. Reference is 10°.



Figure 7. The longitudinal jet autopilot subjected to a step disturbance is shown above with a reference of 10°.



Figure 8. The jet autopilot subjected to ramp disturbance with a reference of 10°.

An attempt to get away without using the rate gyro ended in a simulated jet aircraft disaster. This was done by setting the second gain (k2) to zero, thus "deleting" the rate gyro. Results are shown below.



Figure 9. The jet autopilot with rate gyro removed and no disturbances at a reference of 10°.

### **IV.** Conclusions

The piston aircraft had much simpler aircraft dynamics than the jet powered aircraft, which was to be expected. The jet aircraft had the additional rate gyro, which greatly helped the stability in comparison with the piston aircraft.

The piston aircraft had no error with no disturbance, a steady state error with a step disturbance, and an infinite error with a ramp disturbance.

The jet powered aircraft had no steady state error for both no disturbance and a step disturbance. It also handled the ramp better in the short term, thus possibly allowing the pilot to correct before things get out of control. This was made possible by the rate gyro, without which the jet was unstable. This was seen in figure 9, where without a rate gyro the aircraft quickly went into extreme oscillations.

So although the jet is more complicated, with the addition of a rate gyro it can be controlled better than a simpler piston powered aircraft for which the rate gyro is outside of the budget.